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Power transmission of EHY-2000 – A Hypothesis

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Abstract

Our study concentrates on the high-preciosity type of RF-energy transmission by capacitive coupling (impedance matching). We describe the main fitting parameters, the correct impedance matching pitfalls, and generalize the obtained results for various applications. The careful calculation and the discussion of the matching in detail have particular importance in therapeutical applications because the "load" is the patient who changes by the therapy. A further challenge is that the patients and their treated bodyparts have a broad scale of impedances that forces a frequent complete reset of the matching before starting the treatment. The present calculation gives some clues to optimize the system in a broad impedance regime.

The needs for particular practical application

The energy is used for therapy in oncology (hyperthermia) as a complementary treatment to conventional therapies. During the treatment, the technical challenge is that the RF-generator's maximum power shall be transmitted to the patient during the treatment. This request needs three optimizations:

1. Apply the most effective RF-source to avoid the energy-loss in the source. We chose the E-class amplifier to minimize this loss.
2. The maximum power (best efficacy) of the circuit, when the load resistance ($\underline{Z}_L$) is real (the imaginary part of its impedance is zero), and it is equal with the inner impedance of the source ($\underline{Z}_S$). Conventionally, we choose $\underline{Z}_0 = 50\ \Omega$ as common impedance.
3. Due to the near-field approach in the impedance matching, the conventional fit by minimizing the signal's phase-shift by the load creates the optimal energy transmission. With a careful mechanism instead of the less accurate measurement of the phase-shift, we may use a power meter to measure the forwarded and reflected powers and calculate the high efficacy of transmission.

Transmission Lines – Theory

The applied carrier frequency is $13.56\ \mathrm{MHz}$, chosen as a medical RF-frequency in the radio band that is freely usable for industrial, scientific, and medical (ISM) purposes. The wavelength of this wave in a vacuum is about $\lambda_0 = 22.1\ \mathrm{m}$. The propagation through material shortens the wavelength. For example, the wavelength in a coax cable with $\underline{Z}_0 = 50\ \Omega$ wave impedance at this frequency will be $\lambda_c \approx 13.6\ \mathrm{m}$. The wave impedance denotes the relation between a voltage and a current wave propagating through the cable in forward direction. The wave impedance at all points of the cable depends on frequency but has nearly a constant real value at high frequencies like $13.56\ \mathrm{MHz}$. When the cable is considerable short (about less than one-tenth of the wavelength), the voltage and current amplitude are nearly the same at all points of the cable. For longer cables the voltage and current amplitudes are depending on the location along the cable. In that case, the term "transmission line" is used for the cable.

A source with inner impedance $\underline{Z}_S$ is connected to a transmission line with $\underline{Z}_0$ wave impedance that is terminated by a load $\underline{Z}_L$ – see Fig. 1.

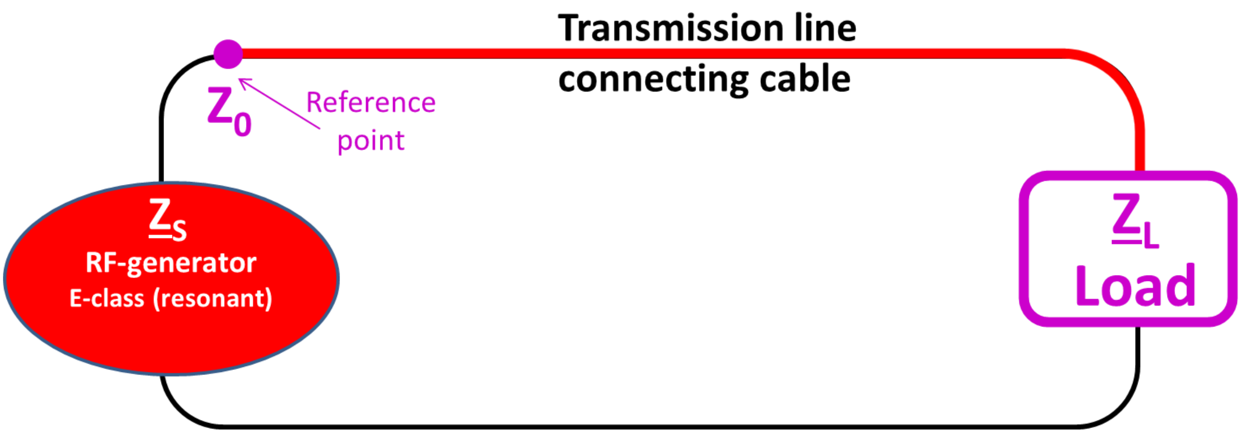

Fig. 1. The simplified drawing of the studied RF circuit.

The generator in the matching case is directly loaded by $\underline{Z}_0$ - equivalent with the tuned multicomponent load. The $\underline{Z}_S$ and $\underline{Z}_0$ are serially connected to each other. The voltage signal travels through the transmission line after switching on the source. The current signal has to be in phase with voltage because of the $\underline{Z}_0 = 50\ \Omega$ resistive wave impedance. Both signals propagate through the cable. The terminating impedance $\underline{Z}_L$ is placed at the end of the cable. If this load differs from the transmission line's wave impedance, the voltage and current suddenly have to jump in their amplitude and phase. Simpler said: Something goes into a port, but something else appears at the port's other end. To bring equilibrium into this equation, we need the reflections. So when a wave is propagating through a transmission line that is terminated by an impedance differing from $\underline{Z}_0$ there will always be reflections.

Imagine that the wave is the first time reflected because the load does not match the wave impedance. The forward and reflected wave overlap. The relation of the overlapped voltage and current waves – in amplitude and phase –describe the transformed load impedance at each point of the cable. In the case that the transmission line has a length of $\lambda_c/2$ and reflections occur due to mismatch, the transformed load impedance at the beginning of the transmission line equals the terminated load impedance. Therefore, this transmission line is also known as 1:1 transformer. From the source' perspective it is connected to the transformed load impedance. If the source impedance $\underline{Z}_S$ does not match the transformed load impedance there will appear a further reflection. This we call re-reflection. The other part that is not re-reflected is transmitted to the generator that can regulate its output signal now. So for mismatching at both ends of the transmission line, there will be several reflections.

Reflections cannot always be avoided and are not a curse in general. In the case of a quarter-wave-transformer two transmission lines with different wave impedances $\underline{Z}_1$ and $\underline{Z}_2$ are connected with a third transmission line – the so-called quarter-wave-transformer. As the name indicates, its length corresponds to one-fourth of the wavelength. Furthermore its wave impedance $\underline{Z}_{\text{qw}}$ has to meet the condition $\underline{Z}_{\text{qw}} = \sqrt{\underline{Z}_1\underline{Z}_2}$. There are reflections – infinity much – that in sum act like a reflection of zero so that the entire power is delivered to the load. Of course, the cable attenuation has to be considered. When a wave propagates through a cable, it will always be attenuated.

In the following, we would like to explain how voltage and current signals propagating through a transmission line can be expressed. We assume a signal with amplitude $y_{\max}$ and phase shift φ at angular frequency ω that only depends on time:

$$y(t) = y_{max} \cdot \cos(\omega t + \varphi) \tag{17}$$

The wave propagation through transmission lines attenuates the signal. So the amplitude exponentially depends on the distance z from the starting reference point in the cable – see Fig. 2.

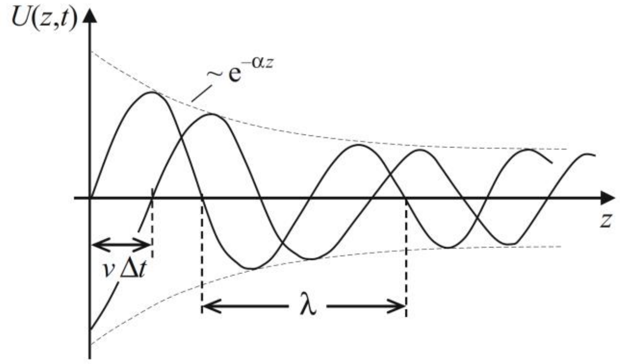

Fig. 2. Attenuated harmonic wave at two points in time with difference Δt [1]

[1] Leone, M., 2018. *Theoretische Elektrotechnik: Elektromagnetische Feldtheorie für Ingenieure*. Wiesbaden: Springer Vieweg, pp. 349-411.

The modified signal with constant attenuation α is:

$$y(t,z) = y_0 e^{-\alpha z} \cdot \cos(\omega t + \varphi) \tag{18}$$

where y_0 is the signal amplitude at the beginning of the transmission line (at $z = 0$ position).
Note that the attenuation constant has to be inserted in the unit Neper $[Np]$.

$$1\ dB = \frac{\ln(10)}{20}\ [Np] \cong 0.115\ Np$$

If we investigate the previous picture, then we see a harmonic signal that is location-dependent. The `frequency` of this signal depends on the wavelength that is shortened when entering the material. The wavelength is linked with the phase constant β, like $\beta = \frac{2\pi}{\lambda}$. The signal can be completed now:

$$y(t,z) = y_0 e^{-\alpha z} \cdot \cos(\omega t - \beta z + \varphi) \tag{19}$$

If we express the signal in complex form, we get the vector $\underline{Y}$:

$$\underline{Y}(t,z) = y_0 e^{-\alpha z} \cdot e^{i(\omega t - \beta z + \varphi)} = y_0 e^{i\omega t} e^{i\varphi} e^{-(\alpha + i\beta)z} = y_0 e^{i(\omega t + \varphi)} e^{-\gamma z} \tag{20}$$

where $\underline{\gamma} = \alpha + i\beta = ik$ is the propagation constant, and k is the wavenumber. The amplitude can be summarized and is complex.

$$\underline{Y}(t,z) = \underline{Y_0}(t,z) e^{-\underline{\gamma} z} \quad \text{where} \quad \underline{Y_0}(t,z) = y_0 e^{i(\omega t + \varphi)} \tag{21}$$

Now we differentiate between forwarded (+) and reflected (-) signal, and the potential (U) and current (I) looks:

$$\begin{aligned} \underline{U}_+ &= \underline{U}_{+0} \cdot e^{-\underline{\gamma} z} & \underline{U}_- &= \underline{U}_{-0} \cdot e^{+\underline{\gamma} z} \\ \underline{I}_+ &= \underline{I}_{+0} \cdot e^{-\underline{\gamma} z} & \underline{I}_- &= \underline{I}_{-0} \cdot e^{+\underline{\gamma} z} \end{aligned} \tag{22}$$

Note that the reflected signal behaves oppositely as the forwarded (mirror imaging at vertical axes) – the negative sign in exponent gets positive.

The sum of the forwarded and reflected signal is the resulting signal.

$$\underline{U}(z) = \underline{U}_+(z) + \underline{U}_-(z) \quad \text{and} \quad \underline{I}(z) = \underline{I}_+(z) + \underline{I}_-(z) \tag{23}$$

$$\underline{U}(z) = \underline{U}_{+0} \cdot e^{-\underline{\gamma} z} + \underline{U}_{-0} \cdot e^{+\underline{\gamma} z} \quad \text{and} \quad \underline{I}(z) = \underline{I}_{+0} \cdot e^{-\underline{\gamma} z} + \underline{I}_{-0} \cdot e^{+\underline{\gamma} z} \tag{24}$$

The wave impedance $\underline{Z}_0$ is:

$$\underline{Z}_0 = \frac{\underline{U}_{+0}}{\underline{I}_{+0}} = \frac{-\underline{U}_{-0}}{\underline{I}_{-0}} \tag{25}$$

The wave impedance for higher frequencies is a constant real value. Then the current can also be expressed by:

$$\underline{I}(z) = \frac{\underline{U}_{+0}}{\underline{Z}_0} \cdot e^{-\underline{\gamma} z} - \frac{\underline{U}_{-0}}{\underline{Z}_0} \cdot e^{+\underline{\gamma} z} \tag{26}$$

The complex reflection $\underline{\Gamma}$ coefficient at an arbitrary point at the transmission line can be calculated as:

$$\underline{\Gamma}(z) = \frac{\underline{U}_-(z)}{\underline{U}_+(z)} = \frac{\underline{U}_{-0}}{\underline{U}_{+0}} \cdot e^{+2\underline{\gamma} z} \tag{27}$$

The reflection coefficient at the end of a transmission line with length l is:

$$\underline{\Gamma}(l) = \frac{\underline{U}_-(l)}{\underline{U}_+(l)} \tag{28}$$

The resulting voltage and current at a terminating impedance $\underline{Z}_L$ have to be the same as the summarized voltage and current at the end of the transmission line.

$$\underline{Z}_L = \frac{\underline{U}(l)}{\underline{I}(l)} = \frac{\underline{U}_+(l) + \underline{U}_-(l)}{\underline{I}_+(l) + \underline{I}_-(l)} = \frac{\underline{U}_+(l) + \underline{U}_-(l)}{\underline{U}_+(l) - \underline{U}_-(l)} \cdot \underline{Z}_0 \tag{29}$$

The reflection coefficient inserted into the previous equation we obtain:

$$\underline{\Gamma}(l) = \frac{\underline{Z}_L - \underline{Z}_0}{\underline{Z}_L + \underline{Z}_0} \tag{30}$$

The absolute value of $\underline{\Gamma}(l)$ ranges in [0,1] interval, where the zero is the perfect matching. In the case of reflection incident and reflected wave interfer and create standing waves. As already meantioned, the transmission line depending on its length acts as an impedance transformer for the load. We would like to derivate the reason for this here. In Fig. 3 a transmission line with wave impedance $\underline{Z}_0$ is terminated by a load impedance $\underline{Z}_{\mathrm{Th}}$.

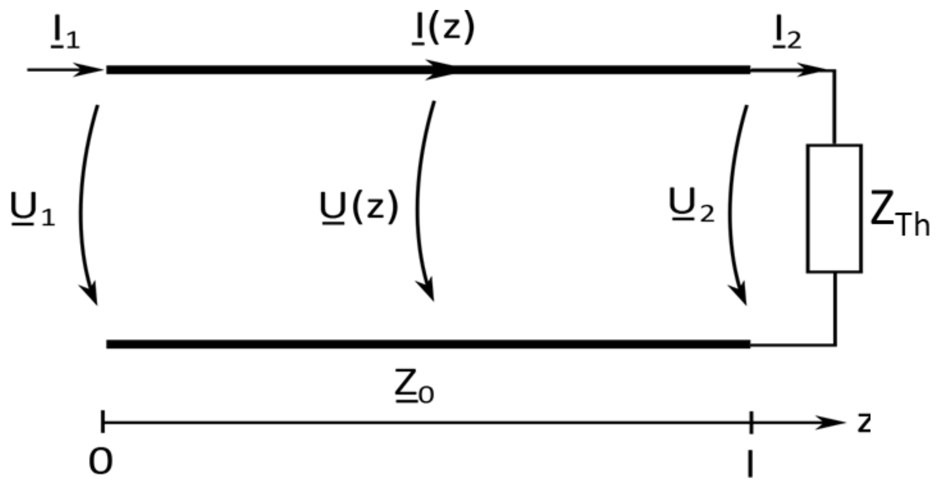

Fig. 3. The load $\underline{Z}_{\mathrm{Th}}$ closes the line at the end of successive drop of voltage

The voltage and current signals can be expressed as the following:

$$\begin{aligned} \underline{U}_1 &= \underline{U}_{+0} + \underline{U}_{-0} & \underline{U}_2 &= \underline{U}_{+0} \cdot e^{-\underline{\gamma}l} + \underline{U}_{-0} \cdot e^{+\underline{\gamma}l} \\ \underline{I}_1 &= \frac{1}{\underline{Z}_0}\left(\underline{U}_{+0} - \underline{U}_{-0}\right) & \underline{I}_2 &= \frac{1}{\underline{Z}_0}\left(\underline{U}_{+0} \cdot e^{-\underline{\gamma}l} - \underline{U}_{-0} \cdot e^{+\underline{\gamma}l}\right) \end{aligned} \tag{31}$$

Forwarded and reflected voltage can then be expressed as:

$$\underline{U}_{+0} = \frac{1}{2}\left(\underline{U}_2 + \underline{Z}_0\underline{I}_2\right)e^{+\underline{\gamma}l} \qquad \underline{U}_{-0} = \frac{1}{2}\left(\underline{U}_2 - \underline{Z}_0\underline{I}_2\right)e^{-\underline{\gamma}l} \tag{32}$$

The voltage and the current at the beginning of the transmission line are then:

$$\begin{aligned} \underline{U}_1 &= \frac{1}{2}\left(\underline{U}_2 + \underline{Z}_0\underline{I}_2\right)e^{+\underline{\gamma}l} + \frac{1}{2}\left(\underline{U}_2 - \underline{Z}_0\underline{I}_2\right)e^{-\underline{\gamma}l} = \underline{U}_2\frac{e^{+\underline{\gamma}l} + e^{-\underline{\gamma}l}}{2} + \underline{Z}_0\underline{I}_2\frac{e^{+\underline{\gamma}l} - e^{-\underline{\gamma}l}}{2} \\ \underline{I}_1 &= \frac{1}{2}\left(\frac{\underline{U}_2}{\underline{Z}_0} + \underline{I}_2\right)e^{+\underline{\gamma}l} - \frac{1}{2}\left(\frac{\underline{U}_2}{\underline{Z}_0} - \underline{I}_2\right)e^{-\underline{\gamma}l} = \frac{\underline{U}_2}{\underline{Z}_0}\frac{e^{+\underline{\gamma}l} - e^{-\underline{\gamma}l}}{2} + \underline{I}_2\frac{e^{+\underline{\gamma}l} + e^{-\underline{\gamma}l}}{2} \end{aligned} \tag{33}$$

Now with the relations, the signals result in the mathematical form of the transmission line equations.

$$\underline{U}_1 = \underline{U}_2 \cosh\left(\underline{\gamma}l\right) + \underline{Z}_0\underline{I}_2 \sinh\left(\underline{\gamma}l\right) \qquad \underline{I}_1 = \frac{\underline{U}_2}{\underline{Z}_0}\sinh\left(\underline{\gamma}l\right) + \underline{I}_2 \cosh\left(\underline{\gamma}l\right) \tag{34}$$

The ratio of these two signals describes the impedance $\underline{Z}_i$ at the beginning of the transmission line.

$$\underline{Z}_i = \frac{\underline{U}_1}{\underline{I}_1} = \frac{\underline{U}_2 \cosh\left(\underline{\gamma} l\right) + \underline{Z}_0 \underline{I}_2 \sinh\left(\underline{\gamma} l\right)}{\frac{\underline{U}_2}{\underline{Z}_0} \sinh\left(\underline{\gamma} l\right) + \underline{I}_2 \cosh\left(\underline{\gamma} l\right)} \tag{35}$$

Note that the load impedance can be expressed by:

$$\underline{Z}_{\mathrm{Th}} = \frac{\underline{U}_2}{\underline{I}_2} \tag{36}$$

We obtain now the transformed load impedance that depends on the load, wavelength and length, and attenuation of the transmission line.

$$\underline{Z}_i = \underline{Z}_0 \frac{\frac{\underline{Z}_{\mathrm{Th}}}{\underline{Z}_0} + \tanh\left(\underline{\gamma} l\right)}{1 + \frac{\underline{Z}_{\mathrm{Th}}}{\underline{Z}_0} \tanh\left(\underline{\gamma} l\right)} \tag{37}$$

In the following, some exceptional cases of impedance transformation by a transmission line are presented. For all cases, the attenuation of the transmission line is assumed to be zero.

1.) $\underline{Z}_{\mathrm{Th}} = \underline{Z}_0$	$\underline{Z}_i = \underline{Z}_0$
2.) $\underline{Z}_{\mathrm{Th}} = 0 \qquad (\tanh(ix) = i \cdot \tanh(x))$	$\underline{Z}_i = i\underline{Z}_0 \cdot \tanh\left(\beta l\right)$
3.) $\underline{Z}_{\mathrm{Th}} = \infty$	$\underline{Z}_i = \frac{\underline{Z}_0}{i \cdot \tanh\left(\beta l\right)}$
4.) $l = \frac{\lambda}{4}$	$\underline{Z}_i = \frac{\underline{Z}_0{}^2}{\underline{Z}_{\mathrm{Th}}}$
5.) $l = \frac{\lambda}{2}$	$\underline{Z}_i = \underline{Z}_{\mathrm{Th}}$

Power Transmission – An Example

In this section, we would like to explain how the power is propagated from the RF generator to the load. The "therapy" load Z_{Th} here represents all that is behind the connecting cable – including the applicator arm, both applicators with boluses, and the patient. In perfect matching $Z_0 = Z_L = \underline{Z}_T + \underline{Z}_{Th}$ the circuit is tuned, and the therapy load $\underline{Z}_{Th}$ completed with the tuners impedance $\underline{Z}_T$ to fix the $Z_0 = Z_L$ requirement. As shown in the previous section, our connecting cable with $\lambda_c/2$ length is a 1:1 impedance transformer. It means that connecting the cable input to the spectrum analyzer shows us the therapy-load impedance Z_{Th} as well – see Fig. 4.

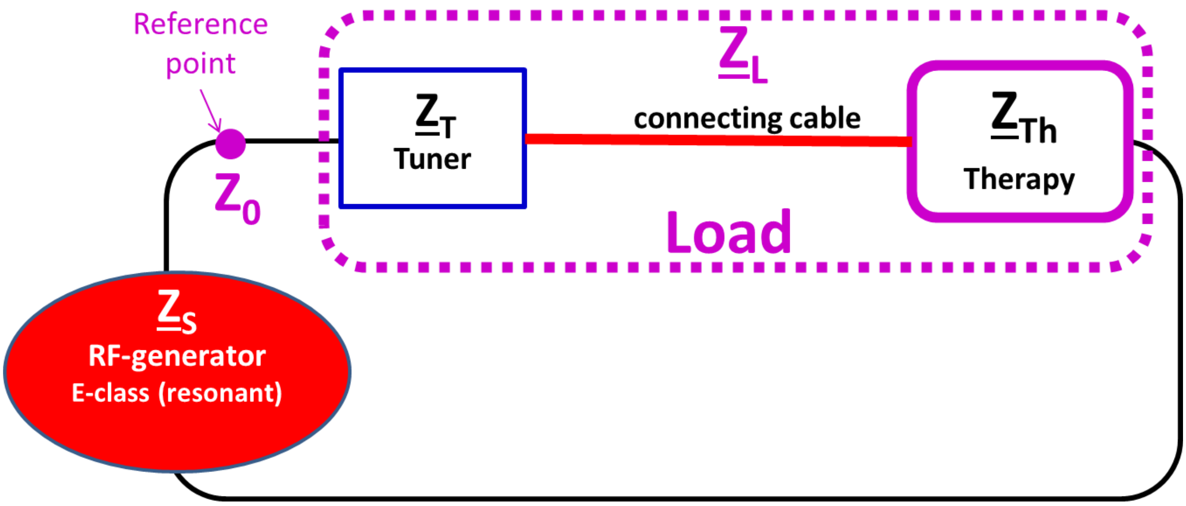

Fig. 4. The RF-circuit with the details discussed in the text

In the following example, the load impedance is assumed to be:

$$\underline{Z}_{\mathrm{Th}} = (20 - 30i)\Omega \tag{38}$$

Further, we want to assume that the insertion loss caused by the tuner is $5\ \%$ and for the connecting cable of $\frac{\lambda_c}{2} = 6.8\ \mathrm{m}$ length is $0.5\ \mathrm{dB}$. The efficacy factor of the tuner is therefore 95 % and for the connecting cable is $89.13\ \%$. (Note the power transmission $\frac{P_{in}}{P_{out}} = 10^{\left(\frac{0.5\ dB}{10}\right)}$)

The coax cable between the RF generator and tuner is assumed to be such short that there are no reflections and the only cable where reflections appear is the connecting cable. For a load $\underline{Z}_{\mathrm{Th}}$ at the end of the connecting cable, the reflections depend on the wave impedance of the cable and load impedance. The reflection coefficient can be calculated as:

$$\underline{\Gamma}_{\mathrm{Th}} = \frac{\underline{Z}_{\mathrm{Th}} - \underline{Z}_0}{\underline{Z}_{\mathrm{Th}} + \underline{Z}_0} \tag{39}$$

The wave impedance is:

$$\underline{Z}_0 = 50\ \Omega \tag{40}$$

It follows:

$$\underline{\Gamma}_{\mathrm{Th}} = -\frac{6}{29} - \frac{15}{29}i \qquad \left|\underline{\Gamma}_{\mathrm{Th}}\right| = 0.5571 \tag{41}$$

Note: The reflection factor describes how much the signal’s voltage and current respectively is reflected by the load. In power consideration, we are interested in the power transmitted to the therapy-load.

$$\frac{P_{\mathrm{refl}}}{P_{\mathrm{forw}}} = \left|\underline{\Gamma}_{\mathrm{Th}}\right|^2 = 0.3103 \tag{42}$$

At the end of the connecting cable are $31.03\ \%$ of power reflected due to the unperfect match between wave and load impedance. The following table shows the wave propagation through the system. At the beginning, we have $100\ \%$ power between generator and tuner. Because of the tuner, only $95\ \%$ of original power is measured between the tuner and connecting cable. Because of the connecting cable loss, only $84.67\ \%$ of the original $100\ \%$ source power is measured between connecting cable and load. Due to the mismatching of impedances, reflections appear at the end of the connecting cable. So that from the original power, only $58.40\ \%$ is transmitted to the load. The other part is reflected. Consider that the load also includes the arm of the therapy applicator where due to radiation further loss appears so that even less power reaches the patient.

The reflected wave propagating through the cable to the tuner is attenuated due to the cable loss. Only $23.41\ \%$ power is measured between the tuner and connecting cable. Now we do not know how much is re-reflected. Let us assume that the treatment is performed when $VSWR = 1.1$. This acceptable value is measured between the generator and the tuner, supposed that the matching is well done. From the VSWR value, we can obtain the reflection coefficient by applying the formula:

$$|\underline{\Gamma}_G| = \frac{VSWR - 1}{VSWR + 1} = \frac{1.1 - 1}{1.1 + 1} = 0.047619 \tag{43}$$

When we start with a forwarded power of $100\ \%$, we can calculate the reflected power with a known reflection factor between the generator and the tuner.

$$\frac{P_{\text{refl}}}{P_{\text{forw}}} = |\underline{\Gamma}_G|^2 = 0.002268 \tag{44}$$

Consequently, obtaining a $VSWR = 1.1$ the reflected power measured between generator and tuner has to be $0.2268\ \%$. We also know that the tuner attenuated this low power. The power between the tuner and connection cable is $0.2387\ \%$. From the $23.41\ \%$ reflected power reaching the beginning of connection cable, only $0.2387\ \%$ can be transmitted; otherwise, there will be a higher $VSWR$ value measured. So, the other $23.1713\ \%$ has to be re-reflected. This is the point where we can calculate the necessary re-reflection factor at the cable beginning at tuner sight (connection cable beginning at the tuner):

$$\frac{P_{\text{refl}}}{P_{\text{forw}}} = |\underline{\Gamma}_T|^2 = 0.9898 \tag{45}$$

This result is for the further calculation of re-reflections used. The re-reflected power propagates through the connection cable. It is attenuated so, and 20.65 % reaches the end of the cable where again it is partly reflected and so on. Fig. 5 visualizes the consecutive power transmission.

Power between generator & tuner	T: transmitted power R: reflected power	Power between tuner & cable	Power between cable & load	T: transmitted power R: reflected power
100 %		95.00 %	84.67 %	T: 58.40 % R: 26.27 %
0.2268 %	T: 0.2387 % R: 23.1713 %	23.41 %		
			20.65 %	T: 14.24 % R: 6.41 %
0.0570 %	T: 0.06 % R: 5.65 %	5.71 %		
			5.04 %	T: 3.48 % R: 1.56 %
0.095 %	T: 0.01 % R: 1.38 %	1.39 %		
			1.23 %	T: 0.85 % R: 0.38 %
…	T: … R: 0.34 %	0.34 %		
			0.30 %	T: 0.21 % R: 0.09 %
		…		

Legend	
	Tuner loss of 5 % -> efficacy 95 %
	Power reflection of 31.03 %
	Cable loss of 0.5 dB for forward and re-reflected waves -> efficacy 89.13 %
	Cable loss of 0.5 dB for reflected waves -> efficacy 89.13 %

Fig. 5. The cascade process of the energy-losses.

Calculating the sum of all power reaching the load:

$$P_{\text{load}} = 58.40\ \% \ + \ 14.24\ \% + 3.48\ \% + 0.85\ \% + 0.21\ \% = 77.18\ \% \tag{46}$$

Now we would like to check the VSWR value between generator and tuner. The forward power is:

$$P_{\text{forw\_GenTun}} = 100\ \% \tag{47}$$

The reflected power is:

$$P_{\text{refl\_GenTun}} = 0.2268\ \% + 0.0570\ \% + 0.0095\ \% = 0.2933\ \% \tag{48}$$

The reflection factor is, therefore:

$$|\Gamma| = \sqrt{\frac{P_{\text{refl\_GenTun}}}{P_{\text{forw\_GenTun}}}} = 0.0542 \tag{49}$$

The VSWR value can be determined by:

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1.11 \tag{50}$$

so our calculation is verified.

Let us suppose we would like to measure the $VSWR$ value between the tuner and connection cable. At this point, we discuss first the power meter itself. The power meter works in a way that a part of forwarded and reflected power is decoupled. It can consist of a short coax cable, and an additional wire is inserted in parallel to the inner conductor. This way, the power meter characterizes the impedance. If we insert it between the tuner and connection cable, the power meter actually extends the connection cable because both have the same wave/characteristic impedance. By measuring the forwarded and reflected waves with a power meter between the tuner and connection cable, we obtain the cable-parameter. Then the measured forwarded power is more than what we originally supposed:

$$P_{\text{forw\_TunCable}} = 95\ \% + 23.1713\ \% + 5.65\ \% + 1.38\ \% + 0.34\ \% = 125.54\ \% \tag{51}$$

The reflected power in this case is:

$$P_{\text{refl\_TunCable}} = 23.41\ \% + 5.71\ \% + 1.39\ \% + 0.34\ \% = 30.85\ \% \tag{52}$$

We see the high forwarded power greater than 100 % and the strong reflection. In the case of a large degree of mismatch, we will measure impossible results. Due to the resulting $U_{\max}$ value, the electrical length of the supply line changes as a result of inductive and capacitive load changes, so the vector diagram of voltage and current also changes. Our measuring instrument determines the power in relation to the wave impedance,

$$P_w = \frac{m}{2Z_0^{'}}\ U_{max}^2 \quad (53)$$

where m is the traveling wave ratio, which is the reciprocal of the SWR, and $Z_0^{'}$ is the wave impedance. With the same power on the line, $U_{\max}$ can be up to $1/\sqrt{m}$ times, which could cause the apparent impossibility.

This effect could be observed experimentally in measuring between the tuner and connection cable. The reflection coefficient and VSWR value are then:

$$|\underline{\Gamma}| = \sqrt{\frac{P_{\text{refl\_TunCable}}}{P_{\text{forw\_TunCable}}}} = 0.4957 \qquad VSWR = \frac{1+|\underline{\Gamma}|}{1-|\underline{\Gamma}|} = 2.97 \quad (54)$$

Power Transmission – Generalization

First, we have to define some quantities.

Quantity	Description	Previous Example
η_C	Power transmission efficacy connection cable	0.8913
η_T	Power transmission efficacy tuner	0.9500
R_L	Power reflection at connection cable end due to load impedance mismatching	0.3103
R_G	Measured/defined power reflection between the generator and tuner	0.002268 $(VSWR = 1.1)$
R_{T_meas}	Measured power re-reflection at connection cable beginning at the tuner[2]	0.4957
R_T	Power re-reflection at connection cable beginning at the tuner[3]	0.9898

The first four constants are given or can be easily calculated. The last two of them have to be determined from formulas we will have constituted at the end of this section. Note that we use for power reflection $|\underline{\Gamma}|^2 = R$. The next step is the generalization of the calculated values.

Coefficient	Description	Previous Example
A_0	Forward power from the generator	$A_0 = 100\ \%$
B_n	Forward power between tuner and bed cable	$B_0 = 95.00\ \%$
C_n	Attenuated forward power at the end of connection cable at the load sight	$C_0 = 84.67\ \%$
D_n	Transmitted power to the load	$D_0 = 58.40\ \%$
E_n	Reflected power at the load	$E_0 = 26.27\ \%$
F_n	Attenuated reflected power at the beginning of connection cable at tuner sight	$F_0 = 23.41\ \%$
G_n	Transmitted power to the tuner	$G_0 = 0.2387\ \%$
H_n	Attenuated transmitted power in backward direction between generator and tuner	$H_0 = 0.2268\ \%$
Note	Re-reflected power becomes the next B – coefficient-> new cycle n	$B_1 = 23.1713\ \%$

[2] It describes the relation of the cumulated re-reflected power (including the initial forwarded power) to the cumulated reflected power.

[3] It describes, how much power is re-reflected fo a single reflected wave.

In the following, the relations between the coefficients are constituted.

$A_0 \cdot \eta_T = B_0$	$\sum_n D_n = P_{load}$
$B_n \cdot \eta_C = C_n$	
$C_n \cdot R_L = E_n$	$\frac{1}{A_0}\sum_n H_n = \mathrm{R}_G$
$C_n \cdot (1 - R_L) = D_n$	
$E_n \cdot \eta_C = F_n$	
$F_n \cdot \mathrm{R}_T = B_{n+1}$	$\frac{\sum_n F_n}{\sum_n B_n} = \mathrm{R}_{T\_meas}$
$F_n \cdot (1 - R_T) = G_n$	
$G_n \cdot \eta_T = H_n$	

The most interesting parameter is, of course, the power transmitted to the load. However, first, we will have a look at what the measured reflection R_{T_meas} is at the cable beginning at tuner sight.

$\mathrm{R}_{T\_meas} = \frac{\sum_n F_n}{\sum_n B_n}$	$\sum_{n=0}^{\infty} F_n = \sum_{n=0}^{\infty} E_n \eta_C = \sum_{n=0}^{\infty} C_n R_L \eta_C = \sum_{n=0}^{\infty} B_n R_L {\eta_C}^2 = R_L {\eta_C}^2 \sum_{n=0}^{\infty} B_n$ $\mathrm{R}_{T\_meas} = \frac{R_L {\eta_C}^2 \sum_{n=0}^{\infty} B_n}{\sum_{n=0}^{\infty} B_n}$ $\mathrm{R}_{T\_meas} = {\eta_C}^2 R_L$

Note that the measured reflection R_{T_meas} between the tuner and connection cable does not correspond to the actual reflection factor R_T at this point. We will determine this factor in the following.

$\mathrm{R}_G = \frac{1}{A_0}\sum_n H_n$	$\frac{1}{A_0}\sum_{n=0}^{\infty} H_n = \frac{1}{A_0}\sum_{n=0}^{\infty} G_n\eta_T = \frac{1}{A_0}\sum_{n=0}^{\infty} F_n\eta_T \cdot (1-R_T) = \frac{1}{A_0}\sum_{n=0}^{\infty} E_n\eta_C\,\eta_T \cdot (1-R_T)$ $= \frac{1}{A_0}\sum_{n=0}^{\infty} C_n\,\eta_C\,\eta_T R_L \cdot (1-R_T)$ $= \frac{1}{A_0}\sum_{n=0}^{\infty} B_n\,{\eta_C}^2\,\eta_T R_L \cdot (1-R_T) = \frac{{\eta_C}^2\,\eta_T R_L \cdot (1-R_T)}{A_0}\sum_{n=0}^{\infty} B_n$ $\sum_{n=0}^{\infty} B_n = \sum_{n=0}^{\infty}\frac{1}{\eta_C}C_n = \sum_{n=0}^{\infty}\frac{1}{\eta_C R_L}E_n = \sum_{n=0}^{\infty}\frac{1}{{\eta_C}^2 R_L}F_n = \sum_{n=0}^{\infty}\frac{1}{{\eta_C}^2 R_L R_T}B_{n+1}$ $B_{n+1} = {\eta_C}^2 R_L R_T \cdot B_n$ $\sum_{n=0}^{\infty} B_n = B_0 + B_1 + B_2 + \cdots$ $B_0\colon A_0\eta_T$ $B_1\colon {\eta_C}^2 R_L R_T \cdot B_0$ $B_2\colon ({\eta_C}^2 R_L R_T)^2 \cdot B_0$ $\sum_{n=0}^{\infty} B_n = A_0\eta_T + \sum_{n=1}^{\infty}({\eta_C}^2 R_L R_T)^n \cdot A_0\eta_T$ ${\eta_C}^2 R_L R_T < 1 \quad \rightarrow \quad \text{apply rule of geometric series}$ $\sum_{n=1}^{\infty}({\eta_C}^2 R_L R_T)^n = \frac{1}{1-{\eta_C}^2 R_L R_T} - 1$ $\sum_{n=0}^{\infty} B_n = A_0\eta_T + A_0\eta_T\left(\frac{1}{1-{\eta_C}^2 R_L R_T} - 1\right) = \frac{A_0\eta_T}{1-{\eta_C}^2 R_L R_T}$ $\frac{1}{A_0}\sum_{n=0}^{\infty} H_n = \frac{{\eta_C}^2\,\eta_T R_L \cdot (1-R_T)}{A_0}\sum_{n=0}^{\infty} B_n = \frac{{\eta_C}^2\,{\eta_T}^2 R_L \cdot (1-R_T)}{1-{\eta_C}^2 R_L R_T}$ convert to R_T … $R_T = \frac{{\eta_C}^2\,{\eta_T}^2 R_L - R_G}{{\eta_C}^2 R_L({\eta_T}^2 - R_G)}$

Of course, we are especially interested in the power delivered to the load.

$P_{load} = \sum_n D_n$	$\sum_{n=0}^{\infty} D_n = \sum_{n=0}^{\infty} C_n \cdot (1-R_L) = \sum_{n=0}^{\infty} B_n\eta_C \cdot (1-R_L) = \eta_C \cdot (1-R_L)\sum_{n=0}^{\infty} B_n$ $\sum_{n=0}^{\infty} B_n = \frac{A_0\eta_T}{1-{\eta_C}^2 R_L R_T}$ $P_{load} = \frac{A_0\eta_C\eta_T \cdot (1-R_L)}{1-{\eta_C}^2 R_L R_T}$

Verification of the generalization - recheck the previous example

After the previous generalization, recheck the power-transmission example calculated before.

Load impedance	$\underline{Z}_L = (20 - 30i)\Omega$
Reflection factor at load	for $\underline{Z}_0 = 50\ \Omega$: $\lvert\underline{\Gamma}_L\rvert = 0.5571$ $R_L = \lvert\underline{\Gamma}_L\rvert^2 = 0.3103$
Power transmission efficacy – connection cable	$\eta_C = 0.8913$
Power transmission efficacy - tuner	$\eta_T = 0.9500$
Reflection factor between generator and tuner	$VSWR = 1.1$ for $\underline{Z}_0 = 50\ \Omega$: $\lvert\underline{\Gamma}_G\rvert = 0.0476$ $R_G = \lvert\underline{\Gamma}_G\rvert^2 = 0.002268$
Delivered power of the generator	$A_0 = 100\ \%$
Measured reflection between tuner and connection cable	$\mathrm{R}_{T\_meas} = 0.2465 \rightarrow VSWR = 2.97$
Reflection factor at the tuner	$R_T = 0.9923$
Transmitted power to the load	$P_{load} = 77.32\ \%$

The power is nearly the same as in the previous example calculation, but here the values are a bit higher because after a lot of reflections, a small power was added. The $VSWR$ value between the tuner and cable is the same for both calculations. The reflection factor at this point is (formula) a bit higher here. Otherwise, the power transmitted back to the generator increases, and therefore the VSWR value was limited too.

Special cases

We will consider the following three special cases:

- no connection cable attenuation
- no reflection at load
- no re-reflection at the tuner

The results:

no connection cable attenuation	$\eta_C = 1.0$ $P_{load} = 94.76\ \%$
no reflection at load	$R_L = 0$ $P_{load} = A_0\eta_C\eta_T = 84.67\ \%$
no re-reflection at the tuner	$R_T = 0$ $P_{load} = A_0\eta_C\eta_T \cdot (1 - R_L) = 58.40\ \%$

The important role of cable attenuation due to multiple reflections looks essential. When the therapy-load matches better to the connection cable, then more power is delivered to the load. In the case that the load does not match the cable, there has to be a strong re-reflection, causing considerable loss of energy.

Conclusion

The matching of the load in the RF-circuit is a complex task. The matching process must fit multiple interconnected parameters, which work collectively and make a sensitive balance of optimum. The therapeutic application's main challenge is the time-dependent load, so the standard fixed antenna matching does not work. The changing patients and their coupling and the changing by the therapy's effect all could drastically modify the tuning conditions, and the coupling worsens, deviates from the optimum. Only the proper real-time adjusting allows long-time stability for the therapeutic efficacy. The above-generalized parametrization makes it possible to follow the situation in real-time. The reaction time is limited by RC time-constant of the circuits, but in the human physiological changes allows a few minutes delay.